

DISSERTATION PROPOSAL

“Cutting planes for Mixed Integer Programming”

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Monday, April 11, 2011

10.30 am

Room: 324 GSIA (West Wing)

My work focuses on cutting planes technology in Mixed Integer Programming. I explore novel classes of valid linear inequalities to strengthen linear relaxations of both Linear and Nonlinear Mixed Integer problems. My dissertation consists of three chapters that investigate theoretically and computationally the families of cuts considered.

The first chapter is based on joint work with Prof. Pietro Belotti and Prof. François Margot. We study linear relaxations of Quadratically Constrained Quadratic Programs. The proposed relaxations are models with both semidefinite constraints (PSD) and linear constraints given by the Reformulation Linearization Technique (RLT). It is known from the literature (Anstreicher, 2007) that PSD and RLT used together yield better bounds than each technique used separately. We adopt a linear outer-approximation of the PSD cone, and we use exclusively linear programming tools to enforce the PSD condition via a cutting plane approach in the lifted space containing the $Y_{ij} = x_i x_j$ variables. We study new classes of valid linear inequalities and we test their effectiveness empirically. These classes include sparse PSD cuts and cuts derived from principal minors. Computational results based on instances from the GLOBALlib and Boxed Constrained Quadratic Programs show that this approach yields better bounds than using solely the standard PSD cuts on top of the RLT inequalities. The C++ code developed for this study has been included in Coin-OR as part of the Couenne project (exact solver for MINLPs).

In the second chapter I present a work closely related to the recent developments in the area of “cuts from multiple rows of the simplex tableau” (Andersen et al., 2007). This chapter is based on joint work with Prof. Egon Balas. We generate intersection cuts from lattice-free convex sets as lift-and-project cuts from multiple-term disjunctions. We use the concept of “Disjunctive Hull” defined for a Mixed Integer Program at a fractional vertex v of its linear relaxation P as the convex hull of points in P satisfying all basic disjunctions that cut off v but no integer point. We examine the relationship between the Disjunctive Hull and the Integer Hull and we give procedures to generate inequalities for the Integer Hull derived from the Cut Generating Linear Program associated to the Disjunctive Hull. Strengthening techniques based on coefficient modularization and monoidal strengthening are also discussed. In this chapter we also analyze the case of 0-1 programming which has not been covered in the literature. My framework applies to this setting with minor changes and produces cuts valid for the 0-1 case but that are not valid for general integer programs. I propose to amplify this chapter by including computational results on general integer and

0-1 problems based on instances from the MIPLIB and using some of the families of cuts subject of this study. Moreover I ran some preliminary experiments separating two families of non facet defining cuts: I consider cuts from fixed shape triangles of type 1 and conic disjunctions having the apex at one vertex of the unit cube and two extreme rays, each containing an additional vertex of the cube. In practice, in order to match the strength of the non facet defining inequalities, a large number of facet defining inequalities is typically needed. I plan to investigate this finding more thoroughly.

In the third chapter I present a theoretical result on strengthening valid inequalities for Mixed Integer Linear Programs. This chapter is also based on joint work with Prof. Egon Balas. There are two distinct strengthening methods for disjunctive cuts; they differ in the way they modularize the coefficients associated to integer constrained variables. We introduce a new variant of one of these methods, the monoidal cut strengthening procedure (Balas and Jeroslow, 1980), based on the paradox that sometimes weakening a disjunction helps the strengthening procedure and results in sharper cuts. We first derive a general result that applies to cuts from disjunctions with any number of terms. It defines the coefficients of the cut in a way that takes advantage of the option of adding new terms to the disjunction. We then specialize this result to the case of split cuts, in particular Gomory Mixed Integer cuts, and to intersection cuts from multiple rows of a simplex tableau. In both instances we give conditions for the new cuts to have stronger coefficients than the cuts obtained by either of the two currently known strengthening procedures.