

DISSERTATION DEFENSE

Vineet Goyal

Combinatorial Optimization under Uncertainty

Tuesday, August 26, 2008

1:30 pm

388 Posner Hall

Most optimization problems in real-life do not have accurate estimates of the problem parameters at the optimization phase. Stochastic optimization models have been studied widely in the literature to model uncertainty in problem parameters but these do not sufficiently guard against the worst-case future in more risk-averse applications. The goal of this thesis is to study optimization approaches under certain models of uncertainty that overcome this shortcoming of a traditional stochastic optimization model.

We consider a general covering problem where the right hand side of the covering constraints (or the demand) is uncertain. For instance, consider a set cover problem where we are given a family of subsets but the set of elements that need to be covered are uncertain. We are interested in constructing a solution such that the worst-case cost over all realizations of uncertainty is minimized. Such a solution can be constructed in one-stage (for problems with one stage of decision making) or multiple-stages (for problems with multiple stages of decision making). We consider one-stage as well as two-stage problems in this model of uncertainty which is referred to as the 'demand-robust model'.

For a two-stage demand-robust problem, we prove a result about the structure of first stage of near-optimal solutions and provide approximation algorithms for specific problems such as Steiner tree, min-cut, minimum multi-cut, vertex cover and facility location in this model. We also develop a 'guess-and-prune' algorithm where we 'guess' the worst case second stage cost which allows us to 'prune' away a set of scenarios which can be completely satisfied within this bound. We use this approach to obtain approximation algorithms for minimum cut and shortest path problems in our model.

The robust optimization approach guards against the worst-case future but tends to be overly conservative if there are some outlier scenarios. To overcome this, we consider a chance-constrained model where we are given a reliability level p and the idea is to select a " p fraction" of the scenarios and find a robust solution on the selected scenarios. The remaining $(1-p)$ fraction of the scenarios are considered as outliers and can be ignored. We show that a general chance-constrained covering problem is at least as hard to approximate as the dense k -subgraph (DkS) problem even when the uncertainty is specified by an explicit list of scenarios. In wake of this hardness, we consider special cases of demand-uncertainty and obtain approximation algorithms for several covering problems in these models.

In both the above models, we consider uncertainty in the right hand side of the constraints. We extend our work to consider uncertainty in the constraint matrix (referred to as 'data uncertainty') and study a chance-constrained knapsack problem where each item has a known deterministic profit but a random size that is distributed according to a known normal distribution independent of the other items. We obtain a polynomial-time approximation scheme for this problem that selects a set of items that satisfy the chance-constraint strictly and achieve near-optimal profit.

In the last chapter, we consider the planning problem for post-disaster logistics where we are interested in opening a set of emergency response centers to provide timely relief to the affected areas after a disaster such as an earthquake. This problem combines aspects of both demand and data uncertainty as both the demand and the underlying transport network depend on the disaster scenario. We develop an efficient sampling-based algorithm to estimate several parameters for a given set of emergency locations such as the fraction of disaster scenarios where all the demand can be covered and average fraction of demand covered across all disaster scenarios. We use the data for the case of Istanbul, Turkey to conduct the computational experiments and present our results.